MATH5835M Practical

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* The MATH5835M module is assessed by an examination (80%) and a practical (20%). This is the practical, worth 20% of your final module mark.
* You must hand in your solution via Gradescope by **Thursday, 14th December 2023, 2pm**.
* Reports must be typeset (not handwritten) and should be no more than 6 pages in length.
* Within reason you may talk to your friends about this piece of work, but you should not send R code (or output) to each other. Your report must be your own work.

## Introduction

The distribution of wind speeds in a city follows a mixture of log-normal distributions: If we denote the wind speed (in miles per hour) by , we have where is random with and the means and standard deviations are given by The density of the log-normal distribution with parameters and is

for all . This function is available in R as dlnorm(). You can generate samples from the wind speed distribution as follows:

wind\_speed <- function(n, w, mu, sigma) {  
 k <- length(w)  
 i <- sample(k, n, replace = TRUE, prob = w)  
 rlnorm(n, mu[i], sigma[i])  
}  
  
w <- c(0.1, 0.4, 0.5)  
mu <- c(2.1, 1.6, -0.5)  
sigma <- c(0.55, 0.7, 0.6)  
wind\_speed(5, w, mu, sigma)

## [1] 0.5922097 9.8571426 10.6458888 1.0282306 0.2759617

## Task 1

We are interested in the probability that the wind speed is larger than 40 miles per hour.

* Use basic Monte Carlo estimation to estimate the probability that the wind speed is larger than 40 miles per hour.
* Determine the root mean squared error of your estimate.

## Task 2

Our aim here is to find a more efficient estimator for the probability from task 1.

* Use importance sampling to estimate the probability that the wind speed is larger than 40 miles per hour.
* Compare the root mean squared error of your estimate with the one from Task 1.

## Task 3

Now, for simplicity, assume that the wind speed follows a single log-normal distribution with parameters and : Furthermore, we assume that is known and is unknown, random and exponentially distributed: .

On five different days we have observed the wind speeds , , , and . Using these data we want to learn about the unknown value . From Bayes rule we know that the conditional distribution of , given the data has density where is the normalising constant, is the density of the distribution, and is the density of the log-normal distribution with parameters and . In Bayesian statistics, is called the *prior density* and is called the *posterior density*.

We want to sample from the posterior density using envelope rejection sampling.

* Produce a plot of as a function of .
* Choose an appropriate density for the proposals for the rejection sampling method. Explain why your choice is appropriate.
* Show a plot of , and from this determine an appropriate value for the constant in the envelope rejection sampling algorithm.
* Implement the envelope rejection sampling algorithm and generate samples from the posterior density . Use these samples to plot a histogram.
* Compare the posterior distribution to the prior distribution, and comment on the differences you see.