MATH5835M Statistical Computing
Exercise Sheet 5

https://www1.maths.leeds.ac.uk/~voss/2023/MATH5835M/
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This does not count towards your final mark, the questions are for self-study only.

Exercise 12. For $\alpha_1, \alpha_2, \alpha_3 \in \mathbb{R}$ consider the matrix

$$P = \begin{pmatrix} 0.4 & \alpha_1 & 0.0 \\ 0.3 & \alpha_2 & 0.6 \\ 0.0 & \alpha_3 & 0.4 \end{pmatrix}.$$

For which values of $\alpha_1, \alpha_2, \alpha_3$ is P a transition matrix? Find a probability vector $\pi \in \mathbb{R}^3$ which is a stationary distribution of the Markov chain with transition matrix P.

Exercise 13. For $\alpha \in (-1, 1)$, consider the AR(1) process given by

$$X_k = \alpha X_{k-1} + \varepsilon_k$$

for all $k \in \mathbb{N}$ and $X_0 = 0$, where $\varepsilon_k \sim \mathcal{N}(0, 1)$ are i.i.d. This process is a Markov Chain with state space $\mathbb{R}$. Write down the transition density $p(x, y)$ and find the stationary density $\pi(x)$ for this Markov Chain.

Exercise 14. Let $(X_n)_{n \in \mathbb{N}_0}$ be a sequence of independent, $\mathcal{N}(0, 1)$-distributed random variables. We can consider this sequence to be a Markov chain. What is the transition density and the density of the initial distribution for this Markov chain?

Exercise 15. Consider the Metropolis-Hastings algorithm with target density

$$\pi(x) = \frac{1}{Z} \sin(x)^2 \exp(-|x|)$$

for all $x \in \mathbb{R}$, where Z is the normalising constants. For the following proposal distributions, determine the formula for the acceptance probability $\alpha(x, y)$:

a) $Y_{n+1} = X_n + \varepsilon_n$ where $\varepsilon_n \sim \mathcal{N}(0, 1)$, independently of $X_0, \ldots, X_n$.

b) $Y_{n+1} = X_n + \varepsilon_n$ where $\varepsilon_n \sim \mathcal{U}[-1, 2]$, independently of $X_0, \ldots, X_n$.

c) $Y_{n+1} \sim \mathcal{N}(0, 1)$, independently of $X_0, \ldots, X_n$.