MATH5835M Statistical Computing
Exercise Sheet 4

https://www1.maths.leeds.ac.uk/~voss/2023/MATH5835M/
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This does not count towards your final mark, the questions are for self-study only.

Exercise 9. In this question, we will derive a Monte Carlo method to estimate the value of an integral over the whole real line. For this, let $f\colon \mathbb{R} \to \mathbb{R}$ be given. Our aim is to estimate $\int_{-\infty}^{\infty} f(x)\,dx$.

a) Let $X_j \sim \mathcal{N}(0, \sigma^2)$ be i.i.d. and

$$\tilde Z_N = \frac{1}{N} \sum_{j=1}^N f(X_j).$$

Determine the expectation $\mathbb{E}\bigl(\tilde Z_N\bigr)$, as an integral.

b) In terms of f, find a function $g\colon \mathbb{R} \to \mathbb{R}$ such that

$$Z_N = \frac{1}{N} \sum_{j=1}^N g(X_j)$$

has expectation $\mathbb{E}\bigl(Z_N\bigr) = \int_{-\infty}^{\infty} f(x)\,dx$.

c) Determine $\mathrm{MSE}(Z_N)$.

Exercise 10. Let $X \sim \mathcal{N}(0,1)$ and $Y \sim \mathcal{N}(3,2)$. A fair coin is tossed, if head shows we define $Z = X$ and otherwise $Z = Y$. Use R to generate 1000 i.i.d. copies of Z and plot a histogram of the values you obtain.

Exercise 11. Let $X_1, X_2, X_3, \ldots \sim \mathrm{Exp}(1)$ be i.i.d. Each X_i is accepted with probability $p(X_i) = \sin(X_i)^2$.

a) Find the density of the accepted samples.

b) Use R to generate 1000 samples from this distribution and plot a histogram of the samples.