MATH5835M Statistical Computing
Exercise Sheet 3

This does not count towards your final mark, the questions are for self-study only.

Exercise 5. [Try to do this exercise without looking up the answer in the book!] Let $X \in \mathbb{R}$ be a random variable and $f, g \colon \mathbb{R} \to \mathbb{R}$ be two functions. Consider the control variates estimator

$$Z_N^{\mathrm{CV}} = \frac{1}{N} \sum_{j=1}^{N} \bigl(f(X_j) - g(X_j)\bigr) + \mathbb{E}\bigl(g(X)\bigr)$$

for $\mathbb{E}\bigl(f(X)\bigr)$. Give a proof that Z_N^{CV} is unbiased and has mean squared error $\mathrm{MSE}\bigl(Z_N^{\mathrm{CV}}\bigr) = \mathrm{Var}\bigl(f(X) - g(X)\bigr)/N$.

Exercise 6. Let $x_0 = 0$ and $x_n = \cos(x_{n-1})$ for all $n \in \mathbb{N}$. Use R to compute x_{20}.

Exercise 7. Let $U \sim \mathcal{U}[0, 1]$ and $X = 1/U^{3/2}$. Create 1000 samples from the distribution of X and create a *meaningful* histogram of your samples.

Exercise 8. The following function is a (failed) attempt to compute

$$\sum_{i=1}^{n-1} (x_{i+1} - x_i)^2,$$

i.e. the sum of squared increments, in R:

```
SomethingWrong <- function(x) {
  n <- length(x)
  sum <- 0
  for (i in 1:n-1) {
    sum <- sum + (x[i+1] - x[i])^2
  }
  return(sum)
}
```

When we apply this function to the vector $(1, 2, 3)$, we do not get the correct answer 2, but `numeric(0)` instead.

```
> SomethingWrong(c(1,2,3))
numeric(0)
```

What is the mistake in the function `SomethingWrong`?