

MATH5835M Statistical Computing

Exercise Sheet 1

<https://www1.maths.leeds.ac.uk/~voss/2023/MATH5835M/>

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This does not count towards your final mark, the questions are for self-study only.

Exercise 1. Let $U \sim \mathcal{U}[0, 1]$ and $X = U^4$.

- a) Compute the exact value of $\mathbb{E}(X)$ analytically.
- b) Compute an estimate for $\mathbb{E}(X)$ using Monte Carlo estimation.

Exercise 2. Let $X \sim \mathcal{N}(0, 1)$ and consider estimating the probability $p = P(\sin(X) > 1/2)$ using a Monte Carlo estimate. How many Monte Carlo samples are required to get an estimate for p which is correct to n decimal places, for $n = 1, 2, 3, 4, 5, 6$? How long would it take to compute these estimates on your computer?

Exercise 3. Let $X, Y \sim \mathcal{N}(0, 1)$. Compute a Monte Carlo estimate for $\mathbb{E}(\max(X, Y))$ which has a relative error of less than 1%. (Hint: The R functions `pmax()` or `ifelse()` can be used to compute the maximum of X_j and Y_j .)